Wings and Bodies," *Journal of Aircraft*, Vol. 9, No. 10, Oct. 1972, pp. 693–702.

[4]Margason, R. J. and Lamar, J. E., "Vortex-Lattice Fortran Program for Estimating Subsonic Aerodynamic Characteristics of Complex Planforms," TN D-6142, 1971, NASA.

[5]Garner, H. C., Hewitt, B. L., and Labrujere, T. E., "Comparison of Three Methods for the Evaluation of Subsonic Lifting-Surface Theory," ARC R & M 3597, June 1968, National Physical Lab., Teddington, England.

[6]James, R. M., "On the Remarkable Accuracy of the Vortex Lattice Discretization in Thin Wing Theory," DAC 67211, Feb. 1969, McDonnell Douglas Co., Long Beach, Calif.

Aerodynamic Loading on High-Speed Ground Vehicles

Peter Crimi* and Ronald A. Johnson†
Avco Systems Division, Wilmington, Mass.

Introduction

DYNAMIC stability and control are of particular concern in the design of high-speed ground vehicles, since propulsion and suspension requirements impose severe limitations on vehicle response to wind gusts and guideway roughness. At commonly considered operating speeds, which are of the order of 300 fps, aerodynamic loading due to vehicle response can be significant, so a reasonably accurate and rapid method for estimating that loading should be useful in vehicle design. A procedure was derived for computing the derivatives of lift, side force and moment coefficients, outlined below. Quasi-steady, small-perturbation flow was assumed. The method accounts for proximity of a ground plane, but does not include the effects of an air cushion suspension, and so is strictly applicable only to magnetically suspended vehicles for estimation of loading due to longitudinal motion. However, the method should provide reasonable estimates of the lateral loading for either type of suspension.

Several studies of related problems have been carried out. Goodman[1] analyzed a slender body of revolution oscillating in a tube. Barrows and Widnall[2] considered a lifting surface near a ground plane or in a tube. Also, Woolard[3] used slender-body theory to determine the loading on the upper surface of vehicles very close to a ground plane.

Problem Formulation

The flow and vehicle motions are referred to coordinates (x,y,z) fixed at the vehicle mass center, the x-axis being coincident with the longitudinal axis. A freestream with velocity of magnitude V and density ρ is directed as indicated in Fig. 1. The mass center is located a distance H above a ground plane, the effect of which can be obtained by placing an image vehicle at $z = -2H$.

Assuming that the vehicle length is much greater than any dimension in the y-z plane, the simplifications afforded by slender-body theory[4] can be employed. It should be noted that the image system must be taken into account in determining the applicability of this assumption. The

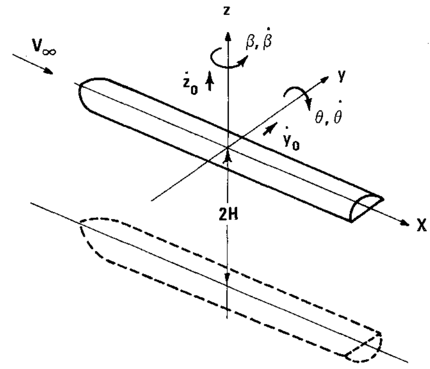

Fig. 1 Coordinate system.

loads per unit of axial length in the y and z direction, respectively, are given, according to the slender-body approximation, by the following integrals evaluated in the crossflow plane:

$$a_y = \int_s (\bar n \cdot \bar j)\Delta P_a ds, \quad a_z = \int_s (\bar n \cdot \bar k)\Delta P_a ds$$

where $\bar n$ is the unit normal to the surface, $\bar j$ and $\bar k$ are unit vectors in the y and z direction, respectively, ds is differential length along the surface in the y-z plane, and

$$\Delta P_a = -\rho V_\infty \frac{\partial}{\partial x}[V_y(x,t)\phi_y(y,z;x) + V_z(x,t)\phi_z(y,z;x)]$$

The functions ϕ_y and ϕ_z are velocity potentials for two-dimensional flow about the vehicle cross section due to translation at unit speed in the negative y and z directions, respectively. The crossflow components V_y and V_z are given by (Fig. 1):

$$V_y = V_\infty\beta - \dot y_0 + x\dot\beta, \quad V_z = V_\infty\Theta - \dot z_0 + x\dot\Theta$$

The specific problem at hand, then, is to find functions ϕ_y and ϕ_z which satisfy Laplace's equation in the crossflow plane, the conditions

$$[(\frac{\partial\phi_y}{\partial y} + 1)\bar j + \frac{\partial\phi_y}{\partial z}\bar k] \cdot \bar n = 0, \quad [\frac{\partial\phi_z}{\partial y}j + (\frac{\partial\phi_z}{\partial z} + 1)\bar k] \cdot \bar n = 0$$

on the surface of the vehicle, and the requirement of flow tangency at the ground plane. Once those functions are

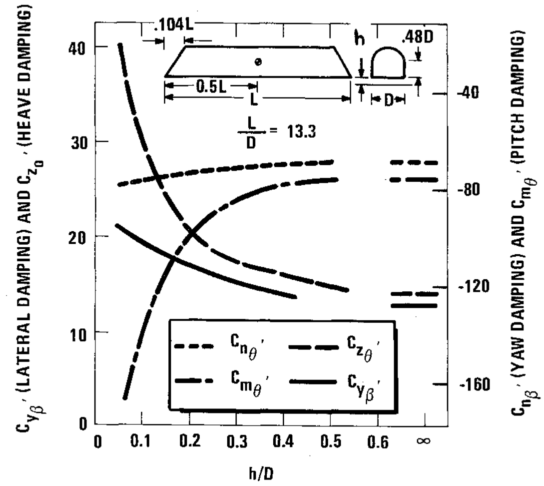

Fig. 2 Rate derivatives of lateral and vertical force and moment coefficients.

Received December 28, 1973; revision received February 27, 1973.

Index categories: Nonsteady Aerodynamics; Airplane and Component Aerodynamics.

*Leader, Special Projects Group. Member AIAA
†Staff Scientist.

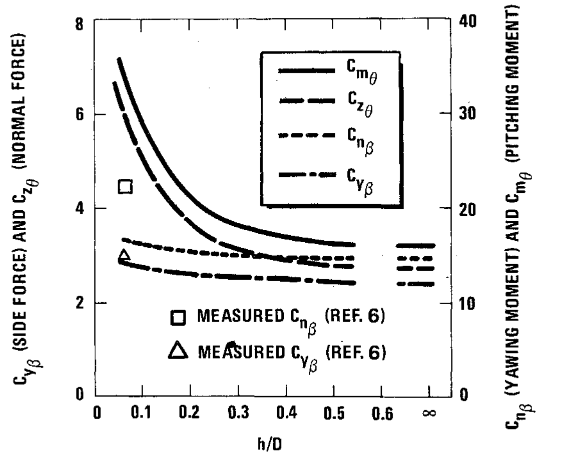

Fig. 3 Displacement derivatives of lateral and vertical force and moment coefficients.

found, a_y and a_z are readily integrated analytically, providing all cross sections are similar, to obtain the vehicle force and moment coefficients.

Method of Solution

Since potential-flow solutions for bodies near a plane, even for elementary geometries, are not readily obtainable in closed form, a modification of the numerical method of Hess and Smith[5] was used to compute ϕ_y and ϕ_z. The solution is cast in the form of a distribution over the surface of source singularities of unknown strength, together with an image distribution. Thus, ϕ_y, for example, is given by

$$\phi_y(y, z;x) = \frac{1}{2\pi} \int_s \sigma_y(s) \ln(r\bar{r})ds$$

where r is the distance from a point (η,ζ) on the surface to the field point (y,z) and $\bar{r}$ is the distance from the corresponding point $(\eta,\zeta -2H)$ on the image surface to (y,z). To approximate the effect of the continuous distribution, the two surfaces are replaced by arrays of N rectilinear elements. The source strength is taken to be constant on each element. Imposition of the flow-tangency condition at the midpoint of each element results in N linear algebraic equations which are solved to obtain the N source strengths.

Results of Computations-Comparison with Experiment

Computations of the derivatives of both lateral and longitudinal force and moment coefficients were carried out for the vehicle geometry sketched in Fig. 2. This configuration was selected because results of measurements of static lateral force and moment coefficients, for this vehicle, near a ground plane, are available from Ref. 6. In the calculations, the base area was assumed to be the maximum cross-sectional area of the vehicle, because it is indicated in Ref. 6 that the flow was separated over the tail during the tests.

The computed variation with gap height of the derivatives of the force and moment coefficients with respect to angular displacement are plotted in Fig. 3. Derivatives with respect to dimensionless angular rates $\theta' = \dot{\theta}D/V$ and $\beta' = \dot{\beta}D/V$ are shown in Fig. 2. Reference area and length are, respectively, maximum cross-sectional area and width of the vehicle.

Ground effect is seen to greatly amplify the longitudinal loading, as would be expected for this configuration. It has a smaller but still significant effect on the lateral loading as well. The aforementioned experimental results for the lateral loading coefficients, obtained for a gap-to-vehicle-width ratio of 0.05, are indicated by the two data points on Fig. 3. The theoretical and experimental moment derivatives are seen to differ by about 30% and the force derivatives by only 3%. While the close agreement of the lateral force coefficients is, perhaps, forfuitous, in light of the uncertainty in the value of effective base area, it can be concluded that the method provides reasonably accurate and rapid estimates of the loading due to response, including the effects of ground proximity.

References

[1]Goodman, T. R., "Aerodynamic Characteristics of a Slender Body Traveling in a Tube," *AIAA Journal,* Vol. 9, No. 4, April 1971, pp. 712–717.

[2]Barrows, T. M. and Widnall, S. E., "Optimum Lift-Drag Ratio for a Ram Wing Tube Vehicle," *AIAA Journal,* Vol. 8, No. 3, March 1970, pp. 491–497.

[3]Woolard, H. W., "Slender-Body Aerodynamics for High-Speed Ground Vehicles," *Journal of Aircraft,* Vol. 8, No. 8, Aug. 1971, pp. 597–602.

[4]Adams, M. C. and Sears, W. R., "Slender-Body Theory-Review and Extension," *Journal of the Aerospace Sciences,* Vol. 20, No. 2, Feb. 1953, pp. 85–98.

[5]Hess, J. L. and Smith, A. M. O., "Calculation of Nonlifting Potential Flow About Arbitrary Three-Dimensional Bodies," *Journal of Ship Research,* Vol. 8, No. 2, Sept. 1964, pp. 22–44.

[6]"A Cost Comparison of Three Tracked Air Cushion Vehicle Configurations," DOT-FR-9, 0032, July 1970, Dept. of Transportation, Washington, D.C.

Normal Mode Solution to the Equations of Motion of a Flexible Airplane

Peter D. Schmitz*

Aeronautical Systems Division, Wright-Patterson Air Force Base, Ohio

Introduction

The equations of motion of a damped linear dynamical system that is idealized by discrete-elements can be expressed by either N second-order Lagrangian equations or by an equivalent set of $2N$ first-order Hamiltonian equations. For small oscillations, the set of N Lagrangian equations can be expressed in the matrix form

$$[m]\{\ddot{q}(t)\} + [c]\{\dot{q}(t)\} + [k]\{q(t)\} = \{F(t)\} \quad (1)$$

where $[m]$, $[c]$, and $[k]$ are $N \times N$ symmetrical inertia, damping and stiffness matrices and where $\{q(t)\}$ and $\{F(t)\}$ are $1 \times N$ column matrices of generalized nodal displacements and equivalent forces, respectively. The equivalent set of $2N$ Hamiltonian equations can be expressed in the partitioned matrix form[1]

$$\begin{bmatrix} [O] & [m] \\ [m] & [c] \end{bmatrix} \begin{bmatrix} \{\ddot{q}(t)\} \\ \{\dot{q}(t)\} \end{bmatrix} + \begin{bmatrix} -[m] & [O] \\ [O] & [k] \end{bmatrix} \begin{bmatrix} \{\dot{q}(t)\} \\ \{q(t)\} \end{bmatrix} = \begin{bmatrix} \{O\} \\ \{F(t)\} \end{bmatrix} \quad (2)$$

where the generalized momenta $\{p\} = [m]\{\dot{q}\}$ are used as a set of auxiliary variables and [0] is an $N \times N$ null matrix.

Received January 17, 1973; revision received March 12, 1973.

Index categories: Structural Dynamic Analysis; Aircraft Vibration.

*Aerospace Engineer, Directorate of Airframe Engineering.